A Control Loop Structure Based on Semi-Nonnegative Matrix Factorization for Input-Coupled Systems

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***Abstract*— This paper introduces a control structure based on the semi-nonnegative matrix factorization (SNMF) to simultaneously control multiple subsystems with a reduced number of inputs. The inspiration for this new control construct is a pin array human machine interface, called Digital Clay. Digital Clay uses a row-column method to control many actuators with a few inputs. The singular value decomposition (SVD) was previously studied to simultaneously control all of the actuators when using the row-column method. However, the SVD technique is not physically implantable in the row-column structure of Digital Clay due to the non-negativity constraints of the control signal. This paper proposes a system based on the SNMF, which is a low-rank approximation method with non-negativity constraints. An SNMF algorithm is presented, and its implementation in a feedback control loop, called the SNMF System, is discussed. Simulation results demonstrate the effectiveness of the SNMF System.**

I. Introduction

Digital Clay is a prototype human-machine interface [1-5]. It allows users to interact tactilely, haptically, and visually with virtual or remote objects. Shown in Fig. 1, it is made up of a square array of pins that are actuated by hydraulic cylinders and uses patented linear position sensors to provide feedback for control and human interaction.

In addition to Digital Clay, other pin arrays have been created [6-15]. The major challenges facing these devices result from the desire for high pin resolution, sufficient force output, fast surface generation, and feasible cost. Due to these criteria, many pin arrays use a row-column input method where the input to each pin is a combination of a row input and a column input [12-15]. The method reduces the number of components, the total device size, and the number of control signals. While the row-column method has been used for pin arrays, it likewise can be implemented in other applications using large numbers of subsystems. The subsystems do not need to be physically arranged in a grid.

For Digital Clay, which has an $n \times n$ array of pins, the n^2

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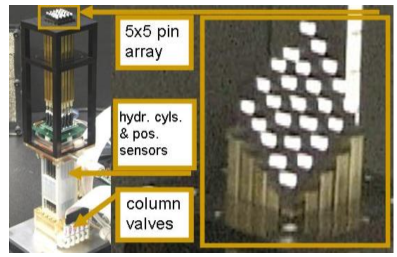

Fig. 1. The Digital Clay prototype (left). A close-up of the array displaying a sloped surface (Right).

cylinders are controlled using $2n$ valves. As shown in Fig. 2, the row valve (C) and the column valve (D) are used to control cylinder (A). The column valve uses compressed air to open and close the control adaptors (B) in its column, thereby controlling the hydraulic flow from the row valve into the cylinder. The extra valve in the bottom left of the diagram is used on the current prototype to connect the row valves to the high and low pressure sources.

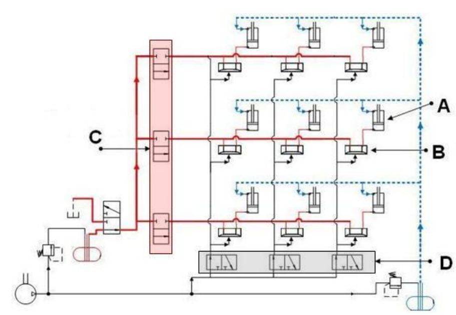

Fig. 2. An abbreviated hydraulic schematic of the current Digital Clay prototype.

One challenge inherent in the row-column method is the coupling of the actuators, creating a loss of simultaneous independent control. As a result, a line scanning pattern, similar to those used for LED arrays, computer monitors, and televisions, has been used to control the pins' position, as shown in Fig. 3. For more on the hydraulic system and the line scanning technique see Zhu and Book [4].

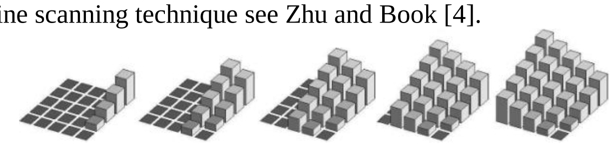

Fig. 3. A sloped surface as produced by the line scanning procedure.

The line scanning procedure has been employed on pin array devices other than Digital Clay [3, 12-13]. The line scanning procedure actuates one column of cylinders at a time, iterating through the columns to generate a surface.

Each new line creates a new intermediate surface. Thus, in an nxn array, only n of the n^2 actuators are employed at any one time. This technique is too slow for many applications and the transients may not be visually appealing. Thus there is a need to reduce the time to generate a surface and to permit all of the cylinders to move simultaneously while maintaining the reduced set of inputs.

In previous work, Winck and Book proposed to control a class of systems that incorporated the row-column method by using singular value decomposition (SVD) [16]. The row-column method can couple the inputs to the cylinders by making them a multiplication of the row and column signals. The SVD was used to reduce the dimension of the control signal in a new control structure called the SVD System. While the SVD System provided significant improvement to the control of systems using the row-column method, it required physical four-quadrant multiplication of the row and column inputs. As will be explained in Section III A, when applied to Digital Clay's hydraulic circuit, this constraint makes the physical realization impractical because the column inputs cannot take negative values. Therefore, in this paper, the semi-nonnegative matrix factorization (SNMF) is used in place of the SVD to obtain the necessary rank-one approximation. The SNMF constrains the column signal to be nonnegative, allowing a direct application to the hydraulic pin array system. The application of the SNMF to the row-column method, called the SNMF System, and its implementation will be described in this paper, along with simulations demonstrating its effectiveness. First, some background is provided on pin array control and the SNMF.

II. Background

A. Previous work on row-column control

In addition to the application of the SVD, there have been a few other attempts to improve on the line scanning procedure used to control an array of actuators. Flemming and Mascaro used a scheduler to select among commands of a row, a column, or a box pattern depending on which combination of configurations was best suited to the required surface [12]. Nakatani et al. used linear programming to select which actuators to command thus improving on the refresh rate over line scanning [13]. The result nevertheless involved a scheduling procedure where a subset of pins was switched on or off. Linear programming was used to provide the optimal actuation order to minimize the surface generation time. Cho et al. also controlled a pin array using a scheduling procedure [14]. Their scheduling procedure involved iterating through every actuator in the grid one at a time. If done quickly enough, relative to the system time constant of the actuators, the control for each actuator worked similarly to pulse-width modulation (PWM). The use of this pseudo-PWM was close to simultaneous control because it moved the actuators in a continuous fashion, but it was still a scheduling procedure in the sense that a single actuator was commanded at a time. The time constant constraint was the main drawback. The speed of response for each actuator and for the whole system was limited by the iteration through nxm actuators, and this constraint becomes multiplicatively greater with increasing array size.

All of these prior methods still involved a scheduling procedure in order to close the loop around each actuator. However, they did improve on the time response relative to line scanning. Each of these techniques was applied to systems using shape memory alloy (SMA) actuators with tight power limitations. To the authors' knowledge, no attempts other than the previous work using the SVD, have been made to close the loop around all of the actuators simultaneously. In this paper, the SNMF is applied to the system as the SVD was applied in previous work. Application of the SNMF extends the use of the approach to systems, such as Digital Clay, with non-negativity constraints on the row or column inputs.

B. Semi-nonnegative matrix factorization

The semi-nonnegative matrix factorization (SNMF) is a low-rank approximation method under non-negativity constraints on one of the low-rank factors [17-18]. Given a matrix $M \in \Re^{nxm}$, the SNMF finds two matrices $W \in \Re^{nxz}$ and $H \in \Re^{mxz}$, such that

$$M \approx WH^T \text{ subject to } H \geq 0, \tag{1}$$

where z<min{n,m} denotes the desired low rank, and H≥0 means that each element of H is nonnegative. Similarly, non-negativity may be imposed on W instead of H.

Imposing non-negativity constraints in the low rank approximation of matrices has been useful in a wide range of applications. The nonnegative matrix factorization (NMF), in which non-negativity is imposed on both of the low rank factors, was popularized by Lee and Seung [19]. They demonstrated that the NMF is able to extract physically meaningful representations using matrices from text documents and facial images. Research on the NMF has been actively conducted both in applications such as bioinformatics and signal processing and in efficient algorithms for its computation [20-23]. Cho et al. used a variant of the NMF in controlling a robotic hand [24]. For the SNMF, in which non-negativity is imposed on only one of the low rank factors, Park and Kim, and Ding et al. studied algorithms and applications in text mining and clustering [17-18]. In this paper, the SNMF is used to reduce the dimension of the control signals in a feedback control loop. To the author's knowledge, the SNMF has not previously been applied in feedback control systems.

III. Application of the SNMF for Feedback Control

A. The row-column method model

The current control structure for Digital Clay and similar pin arrays closes a feedback loop around each cylinder by only commanding motion of a subset of the pins at any one time. A model of the row-column coupling must be obtained to close the loop around all of the pins. The row-column method used in Digital Clay can be modeled as a set of independent subsystems that are coupled by row and column inputs so as to create a set of nonlinear subsystems of the form

$$\dot{x}_{11}(t) = Ax_{11}(t) + br_1(t)c_1(t)$$
$$\dot{x}_{12}(t) = Ax_{12}(t) + br_1(t)c_2(t) \qquad (2)$$
$$\dot{x}_{21}(t) = Ax_{21}(t) + br_2(t)c_1(t) \ \ldots$$
$$\dot{x}_{nm}(t) = Ax_{nm}(t) + br_n(t)c_m(t),$$

where $r_i(t)$ is the i^{th} row input, $c_j(t)$ is the j^{th} column input, and $x_{ij}(t)$ is the state vector of each subsystem coupled nonlinearly by the inputs. In order to visualize the application of the SNMF to this set of subsystems, it is helpful to place the inputs from (2) into a matrix that physically matches the grid defined by the row-column method,

$$\hat{U}(t) = \begin{bmatrix} r_1(t)c_1(t) & \cdots & r_1(t)c_m(t) \\ \vdots & \ddots & \vdots \\ r_n(t)c_1(t) & \cdots & r_n(t)c_m(t) \end{bmatrix}. \qquad (3)$$

$\hat{U}(t)$ is rank-one and can thus be rewritten as an outer product of the vectors of the row and column inputs,

$$\hat{U}(t) = r(t)c^T(t). \qquad (4)$$

Using the hydraulic circuit of Digital Clay, the row input, $r(t) \in \Re^n$, represents the pressure of the fluid in each row. The column input, $c(t) \in \Re^m$, represents the inverse of the resistance provided by the control adaptors in each column. The input to each cylinder, $\hat{U}(t)$, represents a flow command so that the flow into cylinder i,j is

$$\hat{U}_{ij}(t) = Q_{ij}(t) = p_i(t)\frac{1}{R_j(t)} = r_i(t)c_j(t). \qquad (5)$$

For the line scanning procedure, $c_j(t)$ is subject to the constraint $c_j \in \{0,1\}$ and $r_i(t)$ represents a flow command in each row. If $c(t)$ is unrestricted, then $\hat{U}(t)$ can be obtained by the SVD. However, because $c(t)$ represents a resistance when performing simultaneous control with Digital Clay, it must be restricted to $c_j(t) \geq 0$. With this restriction on the column input of Digital Clay, the SVD is not feasibly implementable and the SNMF is used to provide the desired rank-one approximation.

B. The SNMF System

Fig. 4 below shows a block diagram of the SNMF System where each signal, $Y_{des}(t)$, $E(t)$, $U(t)$ and $Y(t) \in \Re^{nxm}$. As previously discussed, with the row-column design the input to the physical system, $\hat{U}(t) = r(t)c^T(t)$, cannot take arbitrary values, and $c_j(t) \geq 0$. However, the output of the controller, $U(t)$, can have arbitrary values and be full rank. Therefore, a rank-one approximation $U(t)$ is generated by the SNMF.

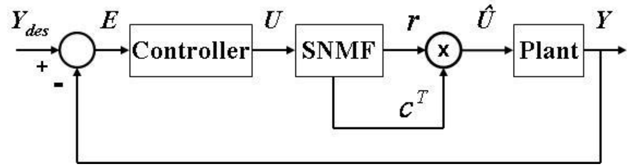

Fig. 4. The SNMF System

The control loop is the same as a classical feedback control structure except that, at each iteration through the feedback loop, the number of control signals is reduced from $n \times m$ to $n+m$ using the SNMF. Then, the rank-one approximation of $U(t)$ is found by the physical row-column multiplication of the row pressures and column resistances: $\hat{U}(t) = r(t)c^T(t)$.

The controller works as follows: as with a classical feedback controller, the desired reference command is compared to the corresponding measured output for every subsystem, and an error matrix, $E(t)$, is generated that is, in general, full rank; a controller designed for the individual subsystems then acts upon each entry of the error matrix and outputs the desired command input matrix $U(t)$; the SNMF of $U(t)$ is found and the pressure and resistance commands generated; these physical signals are multiplied using the row-column method so that the flow command to each cylinder is defined by its corresponding entry in $\hat{U}(t)$; and finally, the response of each of the subsystems is measured and fed back.

C. SNMF algorithm for application to the SNMF System

The purpose of the SNMF in the SNMF System is to obtain a rank-one approximation of the desired input matrix, $U \in \Re^{nxm}$, subject to non-negativity constraints on either the row or the column inputs. The problem can be stated as

$$\min_{r \in \Re^n, c \in \Re^m} \left\| U - rc^T \right\|_F^2 \text{ subject to } c_j \geq 0. \qquad (6)$$

This is a non-convex optimization problem since its objective function is not a convex function. However, an efficient algorithm based on the block coordinate descent method can be developed. Park and Kim proposed to use the block coordinate descent method for a general rank-k approximation [17]. In the rank-one case, the method becomes easier to implement, as described below.

The algorithm proceeds by initializing r with a vector of random real numbers and alternatively solving the following problems until convergence:

$$c^{(k+1)} \leftarrow \arg\min_{c \in \Re^m} \left\| U - r^{(k)} c^T \right\|_F^2 \text{ subject to } c_j \geq 0 \text{, and} \quad (7)$$

$$r^{(k+1)} \leftarrow \arg\min_{r \in \Re^n} \left\| U - r\left(c^{(k+1)} \right)^T \right\|_F^2, \quad (8)$$

where $c^{(k)}$ and $r^{(k)}$ represent the values of c and r at the (k)-th step. These sub-problems have closed form:

$$c^{(k+1)} \leftarrow \left[\frac{U^T r^{(k)}}{\left(r^{(k)}\right)^T r^{(k)}} \right]_+ \text{ and } r^{(k+1)} \leftarrow \frac{U c^{(k)}}{\left(c^{(k)}\right)^T c^{(k)}}. \quad (9)$$

The operator, $[*]_+$, represents a projection operator defined element-wise as

$$\left([c]_+\right)_j = \max\left(c_j, 0\right). \quad (10)$$

The closed form solutions can be efficiently computed because they only involve matrix-vector multiplications: one iteration of the update in (9) costs O(mn) flops. A theoretical property of this SNMF algorithm is as follows. Based on the theory of the block coordinate descent method, every limit point generated by this SNMF algorithm is a stationary point [25]. Although that is a good optimization property, it is weaker than what is offered by the SVD. The SVD, which can be used if the non-negativity constraints are not present, makes it possible to attain the best rank-one approximation from the largest singular value and associated singular vectors. In contrast, the SNMF algorithm typically converges to a local minimum instead of the global minimum. For the SNMF or NMF, no practical algorithm is known to guarantee the global minimum of a low-rank approximation problem. In practice, however, the SNMF algorithm produces useful approximations for feedback control.

IV. Simulation of the SNMF System

The SNMF System was simulated on a range of different size arrays with different subsystem models. The purpose was to demonstrate the effect of the SNMF on the system response and to compare its effect with that of the SVD. To do so, three control techniques were tested: the SNMF System, the SVD System and a system for which every subsystem was controlled independently (IC System). For Digital Clay, the IC System corresponds to using a separate valve to control each cylinder. An example 3x3 array using a model of the Digital Clay hydraulic cylinder is given here.

The model for a Digital Clay cylinder as described by Ngoo and Book was used to test the SNMF System [1]. The input for each subsystem corresponded to a flow command, given by a PWM duty ratio command to a valve. The assumptions of the simulation were: the flow command for the SNMF and SVD Systems corresponded exactly to a multiplication of the row and column commands; there was no variation in the system model from one cylinder to another; and there was no noise present in the system. This enabled a comparison with the SVD System using four-quadrant multiplication and a comparison with the IC System when the row-column multiplication is exact.

A simple PD controller (k_p=1.5 and k_d=1.2) was used for each cylinder as it was sufficient for stability of the IC System and provided appropriate transient response for the tests. For each test a step response is observed. The same commands and control parameters were used for the SNMF, SVD and IC Systems. The responses of the cylinders are denoted by their placement in the array. For example, cylinder 1,2 represents the cylinder in the first row, second column of the array. Except where noted, the initial position of the cylinders in each step response below is 0 mm.

A. Simulation Results and Discussion

First, consider the trivial case of a step response where $Y_{des}(t)$ is a rank-one matrix without any sign changes. In that case, the initial output of the controller, $U(t)$, is also a rank-one matrix with positive factors, so the result of the SNMF algorithm, $\hat{U}(t)$, is equivalent to $U(t)$. Since all of the subsystems are the same, the output, $Y(t)$, is also rank-one. Therefore, the response for the SNMF, SVD, and IC Systems are the same.

A more interesting case occurs for a step response to a matrix of reference commands, $Y_{des}(t)$, with rank greater than one. For example, consider a matrix of random numbers between 0 mm and 50 mm:

$$Y_{des}(t) = \begin{bmatrix} 40.74 & 45.67 & 13.92 \\ 45.29 & 31.62 & 27.34 \\ 6.35 & 4.88 & 47.88 \end{bmatrix} \text{ mm.} \quad (11)$$

$Y_{des}(t)$ is full rank. Its condition number is 10. The response of the SNMF System to this input is shown in Fig. 5. It is different from the response of the IC System. For example, cylinder 2,2 originally responds to a control input that moves the cylinder away from the step command of 31.62 mm. This is not simply overshoot in the classical sense but is due to the rank-one approximation generated by the SNMF System. Although the individual error of cylinder 2,2 is temporarily increased, the controller minimizes the norm of the error matrix measured from all cylinders. Then, at t≈0.7s, the control input from the SNMF System changes the direction of the movement of cylinder 2,2. A similar change occurs at t≈2.6s. These behaviors do not occur in the IC System response and are the primary effects of the SNMF approximation.

Although there are two visible direction changes of cylinder 2,2 in Fig. 5, the control input, shown in Fig. 6 actually changes rapidly. This behavior begins at t≈0.15s. Before that time, the control input for cylinder 2,2 is shown as a descending line without many fluctuations. After t≈0.15s, however, the control input changes at nearly each iteration of the simulation, appearing as dark areas in Fig. 6. Observe that t≈0.15s also corresponds to a visible change in direction of cylinders 3,1, 3,3, and 1,3 in Fig. 5. The change in direction of cylinder 2,2 at t≈0.7s is seen by a change in direction of the control input as highlighted in Fig. 6. These phenomena are caused by rapid changes in the solution to the SNMF approximation. In the first phase of t=0s to t≈0.15s, the best rank-one SNMF solution is distinct, and it is provided as input to the SNMF System except for a couple of instances where the SNMF converges to a local minimum. From t≈0.15s, two or more rank-one solutions alternatively become the best SNMF solution, making rapid oscillations of the control input as shown in Fig. 6. It is these significant changes that can be seen in the response, while the rapid oscillation of the input can be thought of as a PWM between multiple minima found by the SNMF algorithm in that it is filtered out by the systems dynamics. If these input changes do not occur fast enough than they will be seen as low amplitude oscillations in the response of the cylinders.

Now, consider the step response where $Y_{des}(t)$=25*I mm, where I is the 3x3 identity matrix. The responses of cylinders 1,1 and 2,1 for the SNMF System are shown in Fig. 7, in addition to the response of the IC System. The other cylinders on and off the diagonal follow nearly the same path as 1,1 and 2,1 respectively, and so are not shown. For this system, at t=0s there are multiple possible minima for the SNMF approximation and the input changes rapidly between all of these minima throughout the entire response. Notice that although the response appears smooth, it is significantly slower than the IC System due to the rapid changes of the input to each cylinder.

Although the response of the SNMF System is slower than the IC System in Fig. 7, the SNMF System can more quickly generate the entire array than when using line scanning to permit independent control. In Fig. 7, the settling time for the SNMF System is 4.25 seconds as compared to 2.79 seconds for the IC System. However, if line scanning is used to permit independent control, and if it is assumed that the next line starts when the previous line is within 2% of its desired position, then the settling time for the entire surface is 3*2.79 or 8.37 seconds, nearly twice as long as the SNMF System. This example is almost a worst case scenario for the SNMF System because the reference command is full rank with all of its singular values being equivalent. For a lower rank reference command with a greater relative difference between the singular values, the SNMF System will be even faster than line scanning.

The response in Fig. 7 also demonstrates a difference between the SNMF System and the SVD System. The responses of the off-diagonal cylinders of the SVD System remain at 0 mm throughout the entire trajectory because the control input alternates between each cylinder on the diagonal, providing a control input to only one cylinder at a time. This is due to the orthogonality imposed on the SVD solution. However, as seen in Fig. 7, when using the SNMF System, the off-diagonal cylinders move off of their initial positions because the SNMF approximation has no orthogonality constraint. Instead, the minima obtained by the SNMF algorithm move multiple cylinders on the diagonal simultaneously. Thus, to maintain the rank-one constraint, the off-diagonal cylinders also must receive a control input.

A more significant difference between the SVD and SNMF Systems can be seen when some of the cylinders move in the positive direction and some move in the negative direction. An example is given in Fig. 8, where all of the cylinders initially begin at 25 mm and are commanded to (11). Note the slower transient response of the SNMF System compared to that of the SVD System. This is a result of the non-negativity constraint. However, in spite of the differing transient responses, the steady state behavior is the same and the settling time is nearly equivalent, and the SNMF System is still able to generate the surface with no steady-state error.

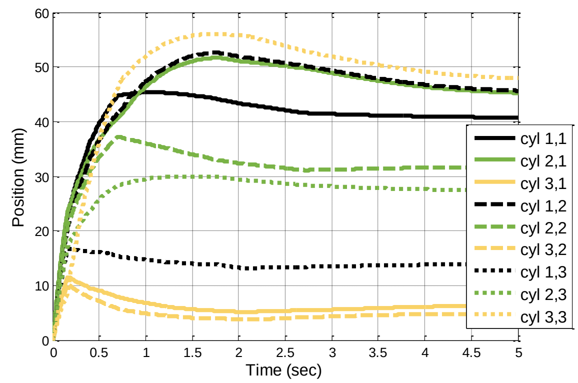

Fig. 5. Step response for all cylinders of $Y_{des}(t)$= random [0,50] mm.

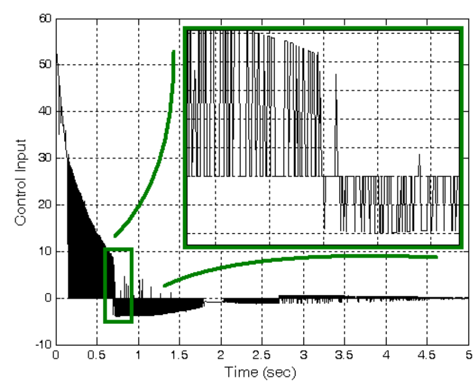

Fig. 6 Control input for cylinder 2,2 for step to $Y_{des}(t)$= random [0,50] mm.

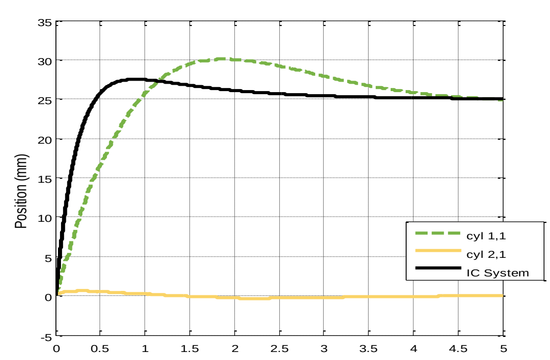

Fig. 7. Step response of cylinders 1,1 and 2,1 of the SNMF System and an IC System cylinder for a step to $Y_{des}(t)$=25*I mm.

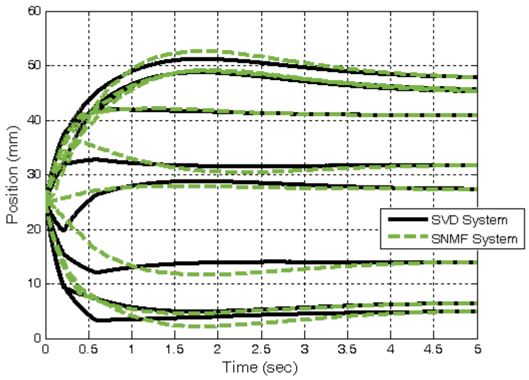

Fig. 8. Step response for all cylinders of the SVD (solid) and SNMF (dashed) Systems to $Y_{des}(t)$= random [0,50] mm with all of the cylinders initially positioned at 25 mm.

V. Conclusions

This paper presents a new control structure that reduces the control inputs from nxm *to* n+m through the use of the SNMF. The SNMF System was motivated by the physical application of Digital Clay, which constrains the column inputs to be nonnegative. However, the application of the SNMF is not limited to Digital Clay, but extends to any system with row-column multiplication and non-negativity constraints.

There is still much work to be done to fully realize the SNMF System. In applying the technique to Digital Clay, the current primary question is the ability to PWM the control adaptors, which is required to create an effective resistance. In addition, pressure sensors are needed to control the pressure in each row. In general, the stability and performance analysis must be theoretically studied. This problem is more difficult with the SNMF than the SVD because linear algebraic theory of the SNMF is less well-understood than that of the SVD. A formal stability analysis using small gain techniques will be the subject of a subsequent paper.

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